MULTIMEDIA UNIVERSITY

STUDENT ID NO

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# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 2, 2015/2016**

## TMA1101 – CALCULUS

(All sections / Groups)

04 MARCH 2016
3:00 p.m. - 5.00 p.m.
(2 Hours)

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**INSTRUCTIONS TO STUDENTS**

1. This question paper consists of six pages with **FIVE** questions.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the answer booklet provided.
4. **You are required to write proper steps.**
5. **No calculators are allowed.**

## QUESTION 1 (10 MARKS)

(a) Find the limits.

***(You must show at least one intermediate step where*** $\lim_{x\to c}$ ***is still needed.)***

(i) $\lim_{x\to 0}\dfrac{(1-x)^2-1}{x}$

(ii) $\lim_{x\to 5}\dfrac{x^2+x-30}{x-5}$ **[2 marks]**

(b) Let $f(x)=\begin{cases}3x+2, & x<4\\ 4x, & x=4\\ x^2-2, & x>4\end{cases}$

(i) Determine $\lim_{x\to 4^-} f(x)$ and $\lim_{x\to 4^+} f(x)$. ***(For this part, you must show at least one intermediate step where*** $\lim_{x\to 4^-}$ or $\lim_{x\to 4^+}$ ***is still needed.)***

(ii) Does $\lim_{x\to 4} f(x)$ exist? Give the reason for your answer.
If it exists, state its value.

(iii) Is the function $f$ continuous at 4? Give the reason for your answer. **[4 marks]**

(c) (i) State the Intermediate Value Theorem (i.e., the full statement including the hypothesis and the conclusion).

(ii) Show that there is a root of the equation $x^4+x-3=0$ in the interval $[0, 2]$.
**[4 marks]**

**Continued ...**

## QUESTION 2 (10 MARKS)

(a) Use the **product rule** or **quotient rule** to find the derivative of $y$.

(i) $y = x^5 \ln x$

(ii) $y = \dfrac{\sin x}{1-4x}$ **[3 marks]**

(b) Use the **chain rule** to find the derivative of $y = (1+\cos\ x)^6$. **[2 marks]**

(c) Given a curve $2x^2y^2 - x = 1$.

(i) Use **implicit differentiation** to find $\dfrac{dy}{dx}$ in terms of $x$ and $y$.

(ii) Determine the gradient of the tangent to the curve at the point $(1,1)$.

**[3 marks]**

(d) Let $f(x) = x^3 + 6x - 10$.

(i) Show that the function $f$ is increasing on the interval $[-2, 2]$

(ii) Find the **absolute minimum** value of $f$ on the interval $[-2, 2]$.

**[2 marks]**

**Continued ...**

## QUESTION 3 (10 MARKS)

(a) (i) What is Euler's formula?

(ii) Use $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$ and $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$ to prove the identity

$$\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x).$$

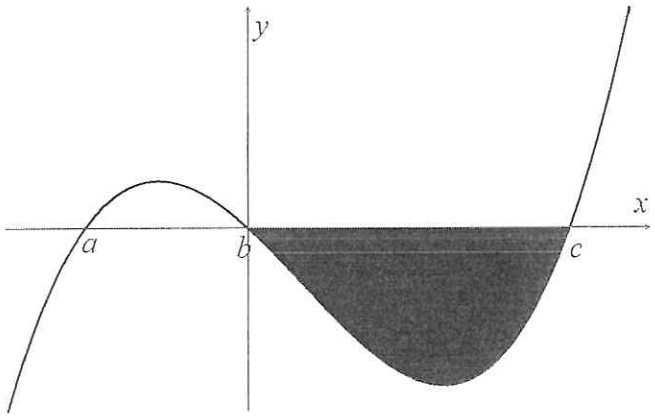
(iii) Hence, find $\int_0^{\pi} \sin^2 x\,dx$. **[4 marks]**

(b) (i) Find the values of $M$ and $N$ in the partial fraction decomposition

$$\frac{x}{x^2+4x+3} = \frac{M}{(x+3)} + \frac{N}{(x+1)}.$$

(ii) Then evaluate $\int \dfrac{x}{x^2+4x+3}\,dx$. **[3 marks]**

(c) The figure shows the graph of $f(x) = x(x+1)(x-2)$ crossing the $x$-axis at $a$, $b$ and $c$.




(i) Find the values of $a$, $b$ and $c$.

(ii) Write down a definite integral that can be used to find the area of the shaded region. Then proceed to find the area. . **[3 marks]**

**Continued ...**

## QUESTION 4 (10 MARKS)

(a) (i) Let the sequence $a_n = \dfrac{2n^3 - 2n^2 + 1}{n^3 + 30n}$. Find $\lim_{n\to\infty} a_n$; you are reminded to show proper steps.

(ii) Determine if the infinite series $\sum_{n=1}^{\infty} \dfrac{2n^3 - 2n^2 + 1}{n^3 + 30n}$ is convergent.

Give the reason for your answer.

**[2.5 marks]**

(b) Given the **geometric series** $\sum_{n=1}^{\infty} ar^{n-1} = 2 + 0.2 + 0.02 + 0.002 + \ldots$,

determine the values of $a$ and $r$. Then find the sum of the series. **[1.5 marks]**

(c) Find the **Maclaurin polynomial** of order 2 for $f(x) = 3e^{-x} - 2x^2$. **[3 marks]**

(d) Let a periodic function $f$ of period $2\pi$ be defined as

$$f(x) = \begin{cases} 1, & -\pi \le x < 0 \\ -1, & 0 \le x < \pi \end{cases}.$$

(i) Sketch the graph of $f$ on the interval $[-\pi, \pi]$.

(ii) The **Fourier series** of $f$ has the form $f(x) = \dfrac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$.

Find $b_3$.

**[3 marks]**

**Continued ...**

## QUESTION 5 (10 MARKS)

(a) Given $F(x,y) = y^2 + 3xy - 2x^2$. Find the partial derivatives $\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial y}$.

**[1 mark]**

(b) Solve the initial value problem involving the first order **separable** differential equation

$\frac{dy}{dx} = \frac{x}{y^2 - 1}$, with $y(2) = 0$. **[2.5 marks]**

(c) Find the general solution $y_h$ of the **homogeneous** differential equations:

(i) $2y''+3y'+y = 0$

(ii) $y''+25 = 0$

**[3 marks]**

(d) A particular solution of the **nonhomogenous** differential equation $y'' - 3y' + 2y = e^x$ is given by $y_p = Bxe^x$.

Determine the value of $B$.

**[3.5 marks]**

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